Design of Experiments

The Bridge to Systematic Innovation

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Introductions

- Name
- Organization
- Job Title/Responsibilities
- Experience in T&E, Combinatorial Testing, DOE, etc.
Agenda

- Some Basic Definitions and Terms
- Various Approaches to Testing Multiple Factors
- Design of Experiments (DOE): a Modern Approach to Combinatorial Testing
- Examples and Demonstration of a DOE
- Using DOE to Achieve Design Optimization
- DOE with Computer Simulation
- Testing a Very Large Number of Factors
- High Throughput Testing
Definition of a Process

A blending of inputs to achieve some desired outputs

Inputs:
- $X_1$
- $X_2$
- $X_3$
- $X_4$
- $X_5$
- $X_6$
- $X_7$

Outputs:
- $Y_1$
- $Y_2$
- $Y_3$
Web-Based Application Process

- CPU Speed
- Type
- RAM Amount
- Type
- HD Size
- Type
- VM
- OS

Performance Tuning

Performance (# home page loads/sec)

Cost ($)

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Combinatorial Test Terminology

Y: Output, response variable, dependent variable

X: Input, factor, independent variable (a measurable entity that is purposely changed during an experiment)

Level: A unique value or choice of a factor (X)

Run: An experimental combination of the levels of the X’s

Replication: Doing or repeating an experimental combination

Effect: The difference or impact on Y when changing X

Interaction: When the effect of one factor depends on the level of another factor
# Performance Tuning Terminology

<table>
<thead>
<tr>
<th>Factors/Inputs (X’s)</th>
<th>Levels (Choices)</th>
<th>Performance/Outputs (Y’s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPU Type</td>
<td>Itanium, Xeon</td>
<td># home page loads/sec</td>
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<tr>
<td>CPU Speed</td>
<td>1 GHz, 2.5 GHz</td>
<td>Cost</td>
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<tr>
<td>RAM Amount</td>
<td>256 MB, 1.5 GB</td>
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<tr>
<td>HD Size</td>
<td>50 GB, 500 GB</td>
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<tr>
<td>VM</td>
<td>J2EE, .NET</td>
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<tr>
<td>OS</td>
<td>Windows, Linux</td>
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</tbody>
</table>

Which factors are important? Which are not? Which combination of factor choices will maximize performance? How do you know for sure? Show me the data.
Graphical Meaning of $\bar{y}$ and $\sigma$

$\bar{y}$ = Average = Mean = Balance Point

$\sigma$ = Standard Deviation

$\approx$ average distance of points from the centerline
Graphical View of Variation

±3σ: Natural Tolerances

Typical Areas under the Normal Curve

-6σ -5σ -4σ -3σ -2σ -1σ 0 +1σ +2σ +3σ +4σ +5σ +6σ

- 68.27%
- 95.45%
- 99.73%
- 99.9937%
- 99.999943%
- 99.9999998%

Simplify, Perfect, Innovate
Approaches to Testing Multiple Factors

• **Traditional Approaches**
  
  • One Factor at a Time (OFAT)
  
  • Oracle (Best Guess)
  
  • All possible combinations (full factorial)

• **Modern Approach**
  
  • Statistically designed experiments (DOE) … full factorial plus other selected DOE designs, depending on the situation
OFAT (One Factor at a Time)

1. Hold $X_2$ constant and vary $X_1$.
   Find the “best setting” for $X_1$.

2. Hold $X_1$ constant at “best setting” and vary $X_2$.
   Find the “best setting” for $X_2$.

3. One factor at a time results

4. One factor at a time results versus optimal results
The Good and Bad about OFAT

• Good News
  • Simple
  • Intuitive
  • The way we were originally taught

• Bad News
  • Will not be able estimate variable interaction effects
  • Will not be able to generate prediction models and thus not be able to optimize performance
Oracle (Best Guess)

X1 = W = Wetting Agent (1=.07 ml; 2=none)
X2 = P = Plasticizer (1=1ml; 2=none)
X3 = E = Environment (1=Ambient Mixing; 2=Semi-Evacuated)
X4 = C = Cement (1=Portland Type III; 2=Calcium Aluminate)
X5 = A = Additive (1=No Reinforcement; 2=Steel)
Y = Strength of Lunar Concrete

<table>
<thead>
<tr>
<th>Run</th>
<th>W</th>
<th>P</th>
<th>E</th>
<th>C</th>
<th>A</th>
<th>Y</th>
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</table>

Does factor C shift the average of Y?
Evaluating the Effects of Variables on Y

What we have is:

\[ E = C \]

What we need is a design to provide independent estimates of effects:

How do we obtain this independence of variables?
All Possible Combinations  
(Full Factorial)

<table>
<thead>
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<th>MATRIX FORM</th>
<th>TREE DIAGRAM</th>
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<tr>
<td>Example 1:</td>
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<tr>
<td>A (2 levels)</td>
<td>A</td>
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<tr>
<td>B (2 levels)</td>
<td>B</td>
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<td>1</td>
<td>1</td>
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<td>2</td>
<td>1</td>
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</table>

| Example 2:  |              |
| A (3 levels) | A            |
| B (3 levels) | B            |
| C (2 levels) | C            |
| 1           | 1            |
| 1           | 2            |
| 1           | 3            |
| 2           | 1            |
| 2           | 2            |
| 3           | 1            |
| 3           | 2            |
| 3           | 3            |
| 1           | 1            |
| 1           | 2            |
| 1           | 3            |
| 2           | 1            |
| 2           | 2            |
| 2           | 3            |
| 3           | 1            |
| 3           | 2            |
| 3           | 3            |
Design of Experiments (DOE)

• An optimal data collection methodology
• “Interrogates” the process
• Used to identify important relationships between input and output factors
• Identifies important interactions between process variables
• Can be used to optimize a process
• Changes “I think” to “I know”
Important Contributions From:

<table>
<thead>
<tr>
<th></th>
<th>TAGUCHI</th>
<th>SHAININ</th>
<th>CLASSICAL</th>
<th>BLENDED APPROACH</th>
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<td>Loss Function</td>
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<td>Emphasis on Variance Reduction</td>
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<td>Response Surface Methods</td>
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</tbody>
</table>

Which bag would a world class golfer prefer?
Statistically Designed Experiments (DOE): Orthogonal or Nearly Orthogonal Designs

- FULL FACTORIALS (for small numbers of factors)
- FRACTIONAL FACTORIALS
- PLACKETT - BURMAN
- LATIN SQUARES
- HADAMARD MATRICES
- BOX - BEHNKEN DESIGNS
- CENTRAL COMPOSITE DESIGNS
- NEARLY ORTHOGONAL LATIN HYPERCUBE DESIGNS

### SIMPLE DEFINITION OF TWO-LEVEL ORTHOGONAL DESIGNS

<table>
<thead>
<tr>
<th>Run</th>
<th>Actual Settings</th>
<th>Coded Matrix</th>
<th>Responses</th>
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<td></td>
<td>(5, 10) 200</td>
<td>(70, 90)</td>
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<td>5 Time</td>
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<td>8</td>
<td>10 Time</td>
<td>90 Temp</td>
<td>200</td>
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</tbody>
</table>
### The Beauty of Orthogonality:
**independent evaluation of effects**

#### A Full Factorial Design for 3 Factors, Each at 2 Levels

<table>
<thead>
<tr>
<th>Run</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>AB</th>
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</table>
Full Factorial vs. Fractional Factorial
(3 factors at 2 levels)

$2^3 = 8$-run Full Factorial Design

$2^{3-1} = 4$-run Fractional Factorial Design
## Screening Design

### Taguchi $L_{12}$ Design

<table>
<thead>
<tr>
<th>Run</th>
<th>1</th>
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</table>
Purposeful changes of the inputs (factors) in order to observe corresponding changes in the output (response).

<table>
<thead>
<tr>
<th>Run</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>$X_4$</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$\ldots$</th>
<th>$\bar{Y}$</th>
<th>$S_Y$</th>
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DOE Helps Determine How Inputs Affect Outputs

i) Factor A affects the average of y

\[ A_1 \quad A_2 \]

\[ y \]

ii) Factor B affects the standard deviation of y

\[ B_1 \quad B_2 \]

\[ y \]

iii) Factor C affects the average and the standard deviation of y

\[ C_1 \quad C_2 \]

\[ y \]

iv) Factor D has no effect on y

\[ D_1 = D_2 \]

\[ y \]
Where does the transfer function come from?

- **Exact transfer Function**
- **Approximations**
  - DOE
  - Historical Data Analysis
  - Simulation
Exact Transfer Functions

• Engineering Relationships
  - $V = IR$
  - $F = ma$

The equation for current ($I$) through this DC circuit is defined by:

$$I = \frac{V}{R_1 \cdot R_2} = \frac{V(R_1 + R_2)}{R_1 \cdot R_2}$$

The equation for magnetic force at a distance $X$ from the center of a solenoid is:

$$H = \frac{NI}{2\ell} \left[ \frac{.5\ell + x}{\sqrt{r^2 + (.5\ell + x)^2}} + \frac{.5\ell - x}{\sqrt{r^2 + (.5\ell - x)^2}} \right]$$

Where
- $N$: total number of turns of wire in the solenoid
- $I$: current in the wire, in amperes
- $r$: radius of helix (solenoid), in cm
- $\ell$: length of the helix (solenoid), in cm
- $x$: distance from center of helix (solenoid), in cm
- $H$: magnetizing force, in amperes per centimeter
Hierarchical Transfer Functions

\[ Y = \text{Gross Margin} = \frac{\text{Gross Profit}}{\text{Gross Revenue}} \]

\[ Y = f(y_1, y_2, y_3, y_4, y_5, y_6) \]

\[ y_1 - y_2 + y_3 + y_4 - y_5 + y_6 \]

\[ \text{Gross Profit} = (\text{Rev}_{\text{equip}} - \text{COG}) + (\text{Rev}_{\text{post sales}} - \text{Cost}_{\text{post sales}}) + (\text{Rev}_{\text{fin}} - \text{Cost}_{\text{fin}}) \]

\[ \text{Cost}_{\text{post sales}} = f(\text{field cost, remote services, suppliers}) \]

\[ x_1 = f(\text{direct labor, freight, parts, depreciation}) \]
Catapulting Power into Test and Evaluation

Statapult® Catapult
The Theoretical Approach
The Theoretical Approach (cont.)

\[ I_0 \ddot{\theta} = r_F F(\theta) \sin \theta \cos \varphi - (M_{r_0} + m_{r_b}) \sin \theta \]

\[ \frac{1}{2} I_0 \dot{\theta}^2 = r_F \int_{\theta_0}^{\theta} F(\theta) \sin \theta \cos \varphi \, d\theta - (M_{r_0} + m_{r_b}) (\sin \theta - \sin \theta_0) \]

\[ \frac{1}{2} I_1 \dot{\theta}_1^2 = r_F \int_{\theta_0}^{\theta_1} F(\theta) \sin \theta \cos \varphi \, d\theta - (M_{r_1} + m_{r_b}) (\sin \theta_1 - \sin \theta_0) \]

\[ x = v_B \cos \left( \frac{\pi}{2} - \theta_1 \right) t - \frac{1}{2} r_B \cos \theta_1 \]

\[ y = r_B \sin \theta_1 + v_B \sin \left( \frac{\pi}{2} - \theta_1 \right) t - \frac{1}{2} g t^2 \]

\[ r_B \sin \theta_1 + (R + r_B \cos \theta_1) \tan \left( \frac{\pi}{2} - \theta_1 \right) - \frac{g}{2v_B^2} \frac{(R + r_B \cos \theta_1)^2}{\cos^2 \left( \frac{\pi}{2} - \theta_1 \right)} = 0. \]

\[ \frac{gl_0}{4r_B \cos^2 \left( \frac{\pi}{2} - \theta_1 \right)} \int_{\theta_0}^{\theta_1} \left[ r_B \sin \theta_1 + (R + r_B \cos \theta_1) \tan \left( \frac{\pi}{2} - \theta_1 \right) \right] \]

\[ = r_F \int_{\theta_0}^{\theta_1} F(\theta) \sin \theta \cos \varphi \, d\theta - (M_{r_0} + m_{r_b}) (\sin \theta_1 - \sin \theta_0). \]
<table>
<thead>
<tr>
<th>Run</th>
<th>A</th>
<th>B</th>
<th>A</th>
<th>B</th>
<th>AB</th>
<th>Y_1</th>
<th>Y_2</th>
<th>Y̅</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>144</td>
<td>2</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td></td>
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<td>+1</td>
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<tr>
<td>3</td>
<td>160</td>
<td>2</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
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</tr>
<tr>
<td>4</td>
<td>160</td>
<td>3</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Avg –

Avg +

Δ
Value Delivery: Reducing Time to Market for New Technologies

INPUT

Pitch <)  (0, 15, 30)
Roll <)  (0, 15, 30)
W1F <)  (-15, 0, 15)
W2F <)  (-15, 0, 15)
W3F <)  (-15, 0, 15)

OUTPUT

Modeling Flight Characteristics of New 3-Wing Aircraft

- Total # of Combinations = $3^5 = 243$
- Central Composite Design: $n = 30$

Patent Holder: Dr. Bert Silich

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Aircraft Equations

\[ C_L = 0.233 + 0.008(P)^2 + 0.255(P) + 0.012(R) - 0.043(WD1) - 0.117(WD2) + 0.185(WD3) + 0.010(P)(WD3) - 0.042(R)(WD1) + 0.035(R)(WD2) + 0.016(R)(WD3) + 0.010(P)(R) - 0.003(WD1)(WD2) - 0.06(WD1)(WD3) \]

\[ C_D = 0.058 + 0.016(P)^2 + 0.028(P) - 0.004(WD1) - 0.13(WD2) + 0.13(WD3) + 0.002(P)(R) - 0.004(P)(WD1) - 0.009(P)(WD2) + 0.016(P)(WD3) - 0.004(R)(WD1) + 0.003(R)(WD2) + 0.20(WD1)^2 + 0.17(WD2)^2 + 0.021(WD3)^2 \]

\[ C_Y = -0.006(P) - 0.006(R) + 0.169(WD1) - 0.121(WD2) - 0.063(WD3) - 0.004(P)(R) + 0.008(P)(WD1) - 0.006(P)(WD2) - 0.008(P)(WD3) - 0.012(R)(WD1) - 0.029(R)(WD2) + 0.48(R)(WD3) - 0.008(WD1)^2 \]

\[ C_M = 0.023 - 0.008(P)^2 + 0.004(P) - 0.007(R) + 0.024(WD1) + 0.066(WD2) - 0.099(WD3) - 0.006(P)(R) - 0.002(P)(WD2) - 0.055(P)(WD3) + 0.023(R)(WD1) - 0.019(R)(WD2) - 0.007(R)(WD3) + 0.007(WD1)^2 - 0.008(WD2)^2 + 0.002(WD1)(WD2) + 0.002(WD1)(WD3) \]

\[ C_{YM} = 0.001(P) + 0.001(R) - 0.050(WD1) + 0.029(WD2) + 0.012(WD3) + 0.001(P)(R) - 0.005(P)(WD1) - 0.004(P)(WD2) - 0.004(P)(WD3) + 0.003(R)(WD1) + 0.008(R)(WD2) - 0.013(R)(WD3) + 0.004(WD1)^2 + 0.003(WD2)^2 + 0.005(WD3)^2 \]

\[ C_e = 0.003(P) + 0.035(WD1) + 0.048(WD2) + 0.051(WD3) - 0.003(R)(WD3) + 0.003(P)(R) - 0.005(P)(WD1) + 0.005(P)(WD2) + 0.006(P)(WD3) + 0.002(R)(WD1) \]
Fusing Titanium and Cobalt-Chrome

Courtesy Rai Chowdhary
Suppose that, in the auto industry, we would like to investigate the following automobile attributes (i.e., factors), along with accompanying levels of those attributes:

A: Brand of Auto: -1 = foreign   +1 = domestic
B: Auto Color: -1 = light 0 = bright +1 = dark
C: Body Style: -1 = 2-door 0 = 4-door +1 = sliding door/hatchback
D: Drive Mechanism: -1 = rear wheel 0 = front wheel +1 = 4-wheel
E: Engine Size: -1 = 4-cylinder 0 = 6-cylinder +1 = 8-cylinder
F: Interior Size: -1 ≤ 2 people 0 = 3-5 people +1 ≥ 6 people
G: Gas Mileage: -1 ≤ 20 mpg 0 = 20-30 mpg +1 ≥ 30 mpg
H: Price: -1 ≤ $20K 0 = $20-$40K +1 ≥ $40K

In addition, suppose the respondents chosen to provide their preferences to product profiles are taken based on the following demographic:

J: Age: -1 ≤ 25 years old +1 ≥ 35 years old
K: Income: -1 ≤ $30K +1 ≥ $40K
L: Education: -1 < BS +1 ≥ BS
### Question:
Choose the best design for evaluating this scenario

### Answer:
$L_{18}$ design with attributes A - H in the inner array and factors J, K, and L in the outer array, resembling an $L_{18}$ robust design, as shown below:

```
  L  K  J
- + - + - + - +
- - + + - - + +
- - - - + + + +
```

| Run* | A | B | C | D | E | F | G | H | y_1 | y_2 | y_3 | y_4 | y_5 | y_6 | y_7 | y_8 | ý  | s |
|------|---|---|---|---|---|---|---|---|-----|-----|-----|-----|-----|-----|-----|----|---|
| 1    | - | - | - | - | - | - | - | - |     |     |     |     |     |     |     |    |   |
| 2    | - | - | 0 | 0 | 0 | 0 | 0 | 0 |     |     |     |     |     |     |     |    |   |
| 3    | - | - | + | + | + | + | + | + |     |     |     |     |     |     |     |    |   |
| 4    | - | 0 | - | - | 0 | 0 | + | + |     |     |     |     |     |     |     |    |   |
| 5    | - | 0 | 0 | 0 | + | + | - | - |     |     |     |     |     |     |     |    |   |
| 6    | - | 0 | 0 | 0 | + | + | - | - |     |     |     |     |     |     |     |    |   |
| 7    | - | + | - | 0 | - | + | 0 | 0 |     |     |     |     |     |     |     |    |   |
| 8    | - | + | 0 | + | 0 | - | + | - |     |     |     |     |     |     |     |    |   |
| 9    | - | + | + | - | 0 | 0 | - | 0 |     |     |     |     |     |     |     |    |   |
| 10   | + | - | 0 | - | - | + | + | 0 |     |     |     |     |     |     |     |    |   |
| 11   | + | - | 0 | - | - | + | + | 0 |     |     |     |     |     |     |     |    |   |
| 12   | + | - | + | 0 | 0 | - | - | + |     |     |     |     |     |     |     |    |   |
| 13   | + | 0 | - | 0 | 0 | - | - | + |     |     |     |     |     |     |     |    |   |
| 14   | + | 0 | 0 | - | + | - | 0 | - |     |     |     |     |     |     |     |    |   |
| 15   | + | 0 | + | - | 0 | + | 0 | - |     |     |     |     |     |     |     |    |   |
| 16   | + | + | - | 0 | + | - | 0 | 0 |     |     |     |     |     |     |     |    |   |
| 17   | + | + | 0 | + | - | 0 | + | + |     |     |     |     |     |     |     |    |   |
| 18   | + | + | + | 0 | - | 0 | + | - |     |     |     |     |     |     |     |    |   |
```

* 18 different product profiles

Segmentation of the population or Respondent Profiles
Modeling The Drivers of Turnover

1. External Market Factors (Local Labor Market Conditions)
   - Local Unemployment Rate
   - Local Employment Alternatives
   - Company’s Market Share

2. Organizational Characteristics and Practices
   - Supervisor Stability
   - Lateral / Upward Mobility
   - Layoff Climate

3. Employee Attributes
   - Time Since Last Promotion
   - Education Level
   - Job Stability History

Process of Deciding to Stay / Leave

Turnover Rate
“From a user’s perspective, a query was submitted and results appear. From Google’s perspective, the user has provided an opportunity to test something. What can we test? Well, there is so much to test that we have an Experiment Council that vets experiment proposals and quickly approves those that pass muster.”

“We evangelize experimentation to the extent that we provide a mechanism for advertisers to run their own experiments.

. . . allows an advertiser to run a (full) factorial experiment on its web page. Advertisers can explore layout and content alternatives while Google randomly directs queries to the resulting treatment combinations. Simple analysis of click and conversion rates allows advertisers to explore a range of alternatives and their effect on user awareness and interest.”

* Taken From: Statistics @ Google in Amstat News, May 2009
• Expected Value Analysis
• Parameter (Robust) Design
• Tolerance Allocation
Expected Value Analysis (EVA)

EVA is the technique used to determine the characteristics of the output distribution (mean, standard deviation, and shape) when we have knowledge of (1) the input variable distributions and (2) the transfer functions.

\[ y_1 = f_1(x_1, x_2, x_3) \]
\[ y_2 = f_2(x_1, x_2, x_3) \]
What is the mean or expected value of the $y$ distribution?

What is the shape of the $y$ distribution?
Parameter Design (Robust Design)

Process of finding the optimal mean settings of the input variables to minimize the resulting dpm.
Parameter Design (Robust Design)

If you’re the designer, which setting for X do you prefer?

Changing the mean of an input may possibly reduce the output variation!
Robust (Parameter) Design Simulation* Example

Controllable:
Plug Pressure (20-50)

Bellow Pressure (10-20)

Ball Valve Pressure (100-200)

Reservoir Level (70-900)

Noise:
Water Temp (70-100)

Nuclear Reservoir Level Control Process

* From SimWare Pro by Digital Computations
Simplify, Perfect, Innovate

Current Date
- n = 139
- Mean = 881.9
- Standard Deviation = 141.6
- LSL = 700
- USL = 900
- C_p = 0.2954
- C_p_k = 0.0427
- DFM = 812.850
- Sg Cap = 1.214

Input Controls

Control Set 1

Plug Pressure (20 to 50)  Bellow Pressure (10 to 20)  Ball Valve Pressure (100 to 200)

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Tolerance Allocation

Which input standard deviations have the biggest effect on the output variation?
In the simple DC circuit shown below, Ohm’s Law says that the total current (I) is equal to \( V / R \), where R is the equivalent resistance of the network.

For this circuit, \[ I = \frac{V}{R_1 \cdot R_2} = \frac{V(R_1 + R_2)}{R_1 \cdot R_2} \]

Suppose

- \( R_1 \) is normally distributed with a mean of 50 Ohms and a standard deviation of 2 Ohms
- \( R_2 \) is normally distributed with a mean of 100 Ohms and standard deviation of 4 Ohms
- the specification limits for current (I) are LSL = .255A and USL = .285A

what is the capability of I?
what current distribution is expected?
### Expected Value Analysis

<table>
<thead>
<tr>
<th>Factor</th>
<th>Distro</th>
<th>First Parameter</th>
<th>Second Parameter</th>
<th>Process Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>Normal</td>
<td>50</td>
<td>2</td>
<td>current</td>
</tr>
<tr>
<td>R2</td>
<td>Normal</td>
<td>100</td>
<td>4</td>
<td># of Simulations</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>StdDev</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Median</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>LSL</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>USL</td>
</tr>
</tbody>
</table>

#### Normal Distro Statistics
- KS Test p-Value (Normal): NA
- dpm: 65,091.811
- Cpk: 0.598
- Cp: 0.616
- Sigma Level: 1.793
- Sigma Capability: 3.013

#### Observed Defect Statistics
- Actual defects: 64,780
- dpm: 64,780.0
- 95% Conf. Inv Lower: 64,298.341
- 95% Conf. Inv Upper: 65,264.182
Expected Value Analysis Example (cont.)

Normal Distribution
Mean = 0.2704
Std Dev = 0.00812

Histogram of current (amps)
Tolerance Allocation Example

Which resistor’s standard deviation has the greater impact on the capability of I?

\[ I = \frac{9(R_1 + R_2)}{R_1 \cdot R_2} \]

LSL = .255
USL = .285
A reduction in $R_1$'s standard deviation (sigma) significantly reduces the dpm while a reduction in $R_2$'s standard deviation has a smaller effect.

### Tolerance Allocation Table

<table>
<thead>
<tr>
<th>Factor</th>
<th>Distro</th>
<th>First Parameter</th>
<th>Second Parameter</th>
<th>current Table (Normal dpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>Normal</td>
<td>50</td>
<td>2</td>
<td>N = 10,000 (in dpm)</td>
</tr>
<tr>
<td>R2</td>
<td>Normal</td>
<td>100</td>
<td>4</td>
<td>R1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>R1</th>
<th>R2</th>
</tr>
</thead>
<tbody>
<tr>
<td>-50% Sigma</td>
<td>2,897</td>
<td>45,852</td>
</tr>
<tr>
<td>-25% Sigma</td>
<td>21,912</td>
<td>53,427</td>
</tr>
<tr>
<td>-10% Sigma</td>
<td>46,150</td>
<td>58,483</td>
</tr>
<tr>
<td>Nominal</td>
<td>63,975</td>
<td>63,438</td>
</tr>
<tr>
<td>+10% Sigma</td>
<td>88,478</td>
<td>69,198</td>
</tr>
<tr>
<td>+25% Sigma</td>
<td>127,102</td>
<td>83,522</td>
</tr>
<tr>
<td>+50% Sigma</td>
<td>196,089</td>
<td>100,553</td>
</tr>
</tbody>
</table>

A reduction in $R_1$'s standard deviation by 50% (from 2 ohms to 1 ohm) combined with an increase in $R_2$'s standard deviation by 25% (from 4 ohms to 5 ohms) results in a dpm = 9,743.

(This result is not shown in the table.)
Growth Rate of Factorial Designs

For 2-level designs and k factors: $2^k$ combinations
• for k = 2 factors: $2^2 = 4$ combinations
• for k = 3 factors: $2^3 = 8$ combinations
• for k = 10 factors: $2^{10} = 1,024$ combinations

For 3-level designs and k factors: $3^k$ combinations
• for k = 2 factors: $3^2 = 9$ combinations
• for k = 3 factors: $3^3 = 27$ combinations
• for k = 10 factors: $3^{10} = 59,049$ combinations

What if the # of factors and/or the number of levels gets large?
Examples of Simulation and High Performance Computing (HPC)

**Power**

Simulation of stress and vibrations of turbine assembly for use in nuclear power generation

**Automotive**

Simulation of underhood thermal cooling for decrease in engine space and increase in cabin space and comfort

**Aerospace**

Evaluation of dual bird-strike on aircraft engine nacelle for turbine blade containment studies

**Electronics**

Evaluation of cooling air flow behavior inside a computer system chassis
Examples of Computer Aided Engineering (CAE) and Simulation Software

Mechanical motion: Multibody kinetics and dynamics
   ADAMS®
   DADS

Implicit Finite Element Analysis: Linear and nonlinear statics, dynamic response
   MSC.Nastran™, MSC.Marc™
   ANSYS®
   Pro MECHANICA
   ABAQUS® Standard and Explicit
   ADINA

Explicit Finite Element Analysis: Impact simulation, metal forming
   LS-DYNA
   RADIOSS
   PAM-CRASH®, PAM-STAMP

General Computational Fluid Dynamics: Internal and external flow simulation
   STAR-CD
   CFX-4, CFX-5
   FLUENT®, FIDAP™
   PowerFLOW®
Preprocessing: Finite Element Analysis and Computational Fluid Dynamics mesh generation
- ICEM-CFD
- Gridgen
- Altair® HyperMesh®
- I-deas®
- MSC.Patran
- TrueGrid®
- GridPro
- FEMB
- ANSA

Postprocessing: Finite Element Analysis and Computational Fluid Dynamics results visualization
- Altair® HyperMesh®
- I-deas
- MSC.Patran
- FEMB
- EnSight
- FIELDVIEW
- ICEM CFD Visual3 2.0 (PVS)
- COVISE
Multidisciplinary Design Optimization (MDO): A Design Process Application

Mastery of interactions between the disciplines (or, subsystems) is as important as the methods & tools used within a single discipline.

Key Elements of MDO:
- Massive Computational Problem;
- Solution by decomposition effective for complex systems;
- Multiprocessor computing simplifies MDO solutions conceptually & enables solutions previously intractable;
- Aids in the management of the design process.

- CFD
- Structures
- Performance
- Controls
- Cost
- NVH
- Safety
- Durability
- Controls Stability
- Cost
Latin Hypercube Sampling

• Method to populate the design space when using deterministic simulation models or when many variables are involved.

• Design space has $k$ variables (or dimensions).
  Ex: Assume $k = 2$

• Suppose a sample of size $n$ is to be taken; stratify the design space into $n^k$ cells.
  Ex: Assume $n = 5$; $n^k = 5^2 = 25$
  Note: there are $n=5$ strata for each of the $k=2$ dimensions.

• Each of the $n$ points is sampled such that each marginal strata is represented only once in the sample.
  Note: each sample point has its own unique row and column.
Applying Modeling and Simulation to Automotive Vehicle Design

**IDENTIFY CTCs, CDPs**

Examples of CTCs:

\[y_1 = \text{weight of vehicle}\]
\[y_2 = \text{cost of vehicle}\]
\[y_3 = \text{frontal head impact}\]
\[y_4 = \text{frontal chest impact}\]
\[y_5 = \text{toe board intrusion}\]
\[y_6 = \text{hip deflection}\]
\[y_7 = \text{rollover impact}\]
\[y_8 = \text{side impact}\]
\[y_9 = \text{internal aerodynamics (airflow)}\]
\[y_{10} = \text{external aerodynamics (airflow)}\]
\[y_{11} = \text{noise}\]
\[y_{12} = \text{vibration (e.g., steering wheel)}\]
\[y_{13} = \text{harshness (e.g., over bumps, shocks)}\]
\[y_{14} = \text{durability (at 100K miles)}\]

Safety CTCs with constraints specified by FMVSS (Federal Motor Vehicle Safety Standards)

No federal requirements on these CTCs

**SCREENING DESIGN (DOE PRO)**

Examples of Critical Design Parameters (CDPs or Xs):

\[x_1 = \text{roof panel material}\]
\[x_2 = \text{roof panel thickness}\]
\[x_3 = \text{door pillar dimensions} \Rightarrow \text{i beam}\]
\[x_4 = \text{shape/geometry}\]
\[x_5 = \text{windshield glass}\]
\[x_6 = \text{hood material, sizing and thickness}\]
\[x_7 = \text{under hood panel material, sizing and thickness}\]

Integrated processes with high fidelity CAE analyses on HPC servers

NASTRAN

RADIOSS

MADYMO

CFD

NASTRAN

RADIOSS

DYNA

MADYMO

The critical few CDP’s
Applying Modeling and Simulation to Automotive Vehicle Design (cont.)

MODELING DESIGN (DOE PRO) → MONTE CARLO SIMULATION (DFSS MASTER) → VALIDATION

CDPs, CTPs → Robust Designs

NASTRAN, RADIOSS, MADYMO
High Fidelity Models
Response Surface Models
Low Fidelity Models
NASTRAN, RADIOSS, MADYMO
High Fidelity Models
Summary of "Modeling the Simulator"

1. Perform Screening Design Using the Simulator if necessary
   - Critical Parameters ID'd
2. Perform Modeling Design Using the Simulator to Build Low Fidelity Model
   - Transfer Function on Critical Parameters
3. Perform Expected Value Analysis, Robust Design, and Tolerance Allocation Using Transfer Function
   - Optimized Design
4. Validate Design Using the Simulator
   - Optimized Simulator
5. Build Prototype to Validate Design in Real World

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Environments Where Simulation and Modeling Is Beneficial

- A high number of design variables
- A substantial number of design subsystems and engineering disciplines
- Interdependency and interaction between the subsystems and variables
- Multiple response variables
- Need to characterize the system at a higher level of abstraction
- Time and/or space must be compressed
Introduction to High Throughput Testing (HTT)

- A recently developed technique based on combinatorics
- Used to test myriad combinations of many factors (typically qualitative) where the factors could have many levels
- Uses a minimum number of runs or combinations to do this
- Software (e.g., ProTest) is needed to select the minimal subset of all possible combinations to be tested so that all 2-way combinations are tested.
- HTT is not a DOE technique, although the terminology is similar
- A run or row in an HTT matrix is, like DOE, a combination of different factor levels which, after being tested, will result in a successful or failed run
- HTT has its origins in the pharmaceutical business where in drug discovery many chemical compounds are combined together (combinatorial chemistry) at many different strengths to try to produce a reaction.
- Other industries are now using HTT, e.g., software testing, materials discovery, integration, and verification testing (see example on next page).
We would like to perform verification testing with 4 input factors described below.

All possible combinations would involve how many test combinations?

If we were interested in testing all pairs only, how many runs would be in the test matrix and what would those combinations be? To answer this question, we used our ProTest software. See next page.

<table>
<thead>
<tr>
<th>Sensor Type</th>
<th>Weapon Type</th>
<th>External Data Link</th>
<th>Target Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>W1</td>
<td>Yes</td>
<td>T1</td>
</tr>
<tr>
<td>S2</td>
<td>W2</td>
<td>No</td>
<td>T2</td>
</tr>
<tr>
<td>S3</td>
<td>W3</td>
<td></td>
<td>T3</td>
</tr>
<tr>
<td>S4</td>
<td></td>
<td></td>
<td>T4</td>
</tr>
</tbody>
</table>

一页
## High Throughput Testing Example (cont)

### 20 Test Cases

<table>
<thead>
<tr>
<th>Case</th>
<th>Sensor</th>
<th>Weapon</th>
<th>Data Link</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>S1</td>
<td>W2</td>
<td>Yes</td>
<td>T1</td>
</tr>
<tr>
<td>2</td>
<td>S4</td>
<td>W1</td>
<td>Yes</td>
<td>T2</td>
</tr>
<tr>
<td>3</td>
<td>S2</td>
<td>W1</td>
<td>No</td>
<td>T3</td>
</tr>
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<td>4</td>
<td>S3</td>
<td>W3</td>
<td>Yes</td>
<td>T4</td>
</tr>
<tr>
<td>5</td>
<td>S2</td>
<td>W3</td>
<td>Yes</td>
<td>T5</td>
</tr>
<tr>
<td>6</td>
<td>S4</td>
<td>W3</td>
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HTT: Pairwise Testing Optimization

- Suppose we had 75 Factors to test.
- Suppose we wanted to test each of these at 2 levels.
- How many total combinations are there?

\[ 2^{75} = 37,778,931,862,957,161,709,568 \]

i.e., 37 Sextillion, 778 Quintillion, 931 Quadrillion, 862 Trillion, 957 Billion, 161 Million, 709 Thousand, 568

- What is the minimum number of these combinations that will have to be tested in order to test every 2-way combination?

- To answer this question, we used our Pro-Test software. The answer is 14 runs or experimental combinations.

- For k factors each having the same number of levels tested, say v, then the minimum number of tests \( \approx v^2 (\ln k) \)
HTT Applications

- Reducing the cost and time of testing while maintaining adequate test coverage
- Integration, functionality, and verification testing
- Creating a test plan to stress a product and discover problems
- Prescreening before a large DOE to ensure all 2-way combinations are feasible before discovering, midway through an experiment, that certain combinations are not feasible
- Developing an “outer array” of noise combinations to use in a robust design DOE when the number of noise factors and settings is large
Key Take-Aways

- Various approaches to combinatorial test, to include OFAT and Oracle.
- DOE brings orthogonal or nearly orthogonal designs into play.
- Orthogonality (both vertical and horizontal balance in a design) is key to being able to evaluate the effects of factors and their interactions independently from one another.
- Factorial designs are great, but in a world of large test design spaces, we need something else.
- Nearly Orthogonal Latin Hypercube Designs provide a sampling strategy to test a large number of factors with a much smaller number of runs than what a factorial design requires, while still retaining adequate orthogonality. These are particularly useful when designing experiments for computer simulations.
- High Throughput Testing is a way to get great test coverage (i.e., all pairwise combinations) with a minimal number of runs. This would be a candidate design for OT&E when we are trying to verify and validate performance in an operational envelope.
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